Indian Statistical Institute, Bangalore
B. Math (Hons.) Third Year
First Semester - Optimization

Back Paper Exam   Date: January 04, 2019
Maximum marks: 50   Duration: 3 hours

Each question carries 5 marks

1. Prove that for a matrix A, there exists a invertible matrix G such that GA is upper echelon.

2. Find the QR decomposition of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

3. Prove the singular values are unique.

4. If $A \geq 0$ is irreducible, prove that $I + A$ is primitive.

5. Find Perron pair for $\begin{pmatrix} 0 & 1 & 3 \\ 3 & 0 & 3 \\ 0 & 2 & 0 \end{pmatrix}$.

6. Find least square solution to the system $x_1 - x_2 = 2$; $x_1 + x_2 = 4$; $2x_1 + x_2 = 8$.

7. Prove that the dual of dual is primal.

8. State and prove minmax Theorem.

9. Solve
$$\begin{aligned} \text{maximize} \quad & 24x_1 + 36x_2 \\ \text{subject to} \quad & 40x_1 + 80x_2 \leq 560 \\ & 6x_1 + 8x_2 \leq 72 \\ & x \geq 0. \end{aligned}$$

10. (a) Solve the matrix games $A = I_n$.

    (b) Solve the matrix game $\begin{pmatrix} 7 & 8 \\ 6 & 7 \end{pmatrix}$ *(Marks: 2)*.